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Geometric Stabilization of Non-Abelian Braids Under Noise via the Geometric Schrodinger Equation

Jeremiah D Pope

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Abstract

We present a numerical investigation of non-Abelian braid execution under stochastic perturbations using the Geometric Schrodinger Equation (GSE). Unlike conventional topological quantum computing approaches that rely on abstract braid equivalence and idealized adiabatic assumptions, the GSE introduces a deterministic geometric flow with an auxiliary stabilization channel. We demonstrate that braid execution can be realized as a dynamically attracting process rather than a fragile path-following procedure. Using an explicit braid word and continuous noise injection, we show bounded planar phase motion and exponential convergence in the lift dimension. These results indicate that topological equivalence can emerge as a geometric attractor, providing a control-theoretic foundation for robust braid execution in realistic noisy environments.

Introduction

Topological approaches to quantum computation propose encoding information in global properties of particle trajectories, most notably the braiding of non-Abelian anyons. In idealized models, braid operations are assumed to be intrinsically fault-tolerant because local perturbations cannot alter global topological classes. However, practical implementations face significant challenges: control noise, finite temperature effects, non-adiabatic transitions, and imperfect device geometries all introduce deviations from the idealized braid paths.

This work addresses a core limitation of standard topological quantum computing frameworks: the absence of a dynamical stabilization mechanism. Traditional treatments classify braids algebraically but do not specify how physical systems are dynamically driven toward a desired braid class in the presence of noise. The Geometric Schrodinger Equation (GSE) offers an alternative perspective by embedding phase evolution within a deterministic geometric flow augmented by a stabilizing lift dimension. In this framework, braid execution is not merely symbolic but realized as an attracting trajectory in an extended state space.

Braids and Non-Abelian Operations

In braid group language, a braid is represented as a word composed of generators σ_i and their inverses. For non-Abelian anyons, such braid words correspond to unitary operations acting on a computational Hilbert space. While topological invariance protects the abstract equivalence class, the physical realization of the braid requires continuous control of system parameters over time.

Limitations of Static Topological Protection

Standard approaches assume that as long as a braid is not topologically altered, errors remain benign. In practice, noise can cause leakage, unintended excitations, or deviations that are not automatically corrected by topology alone. This motivates the need for a dynamical mechanism that actively suppresses deviations and forces convergence toward the intended braid outcome.

The Geometric Schrodinger Equation

The GSE replaces the complex unit with a real geometric operator that induces planar rotation combined with lift and damping in an auxiliary dimension. The resulting dynamics resemble a damped helical flow: phase-like motion persists in the plane, while deviations are absorbed by the lift channel and relaxed via damping. This structure naturally suggests a mechanism for stabilizing operations that would otherwise rely on idealized assumptions.

Model and Methods
State Space and Variables

We consider a state vector

$$\Phi(t) = (u(t), v(t), V(t)),$$

where (u,v) represent planar phase components and V is a lift variable responsible for stabilization. The planar subsystem undergoes rotational dynamics analogous to unitary phase evolution, while the lift dimension introduces controlled dissipation.

Dynamical Equations

The evolution equations consist of:

- planar rotation with orientation determined by the braid generator,
- lift proportional to planar radius,
- linear damping in the lift channel.

Stochastic noise is injected continuously into all channels to model realistic perturbations. Importantly, no noise suppression or error correction is applied externally; stability, if present, must arise from the intrinsic dynamics.

Braid Specification

We study the explicit braid word

$$\beta = \sigma_1 \, \sigma_2^{-1} \, \sigma_1 \, \sigma_2 \, \sigma_1 \, \sigma_2.$$

Each generator is implemented as a finite-duration control segment, with orientation determining the sign of planar rotation. This braid is nontrivial and representative of challenging operations in non-Abelian anyon models.

Numerical Experiment

Simulation Setup

The system is initialized at (u,v,V) = (1,0,0). Each braid segment is executed over a fixed time interval, with continuous Gaussian noise applied to all state variables. Parameters are chosen such that the planar dynamics remain neutrally stable while the lift channel provides dissipative stabilization.

Observables and Diagnostics

We monitor:

- the planar trajectory (u(t),v(t)),
- the instantaneous radius $r(t) = \sqrt{u^2 + v^2}$,
- the lift variable V(t).

Stability is assessed via boundedness of the planar radius and convergence of V to a steady-state value.

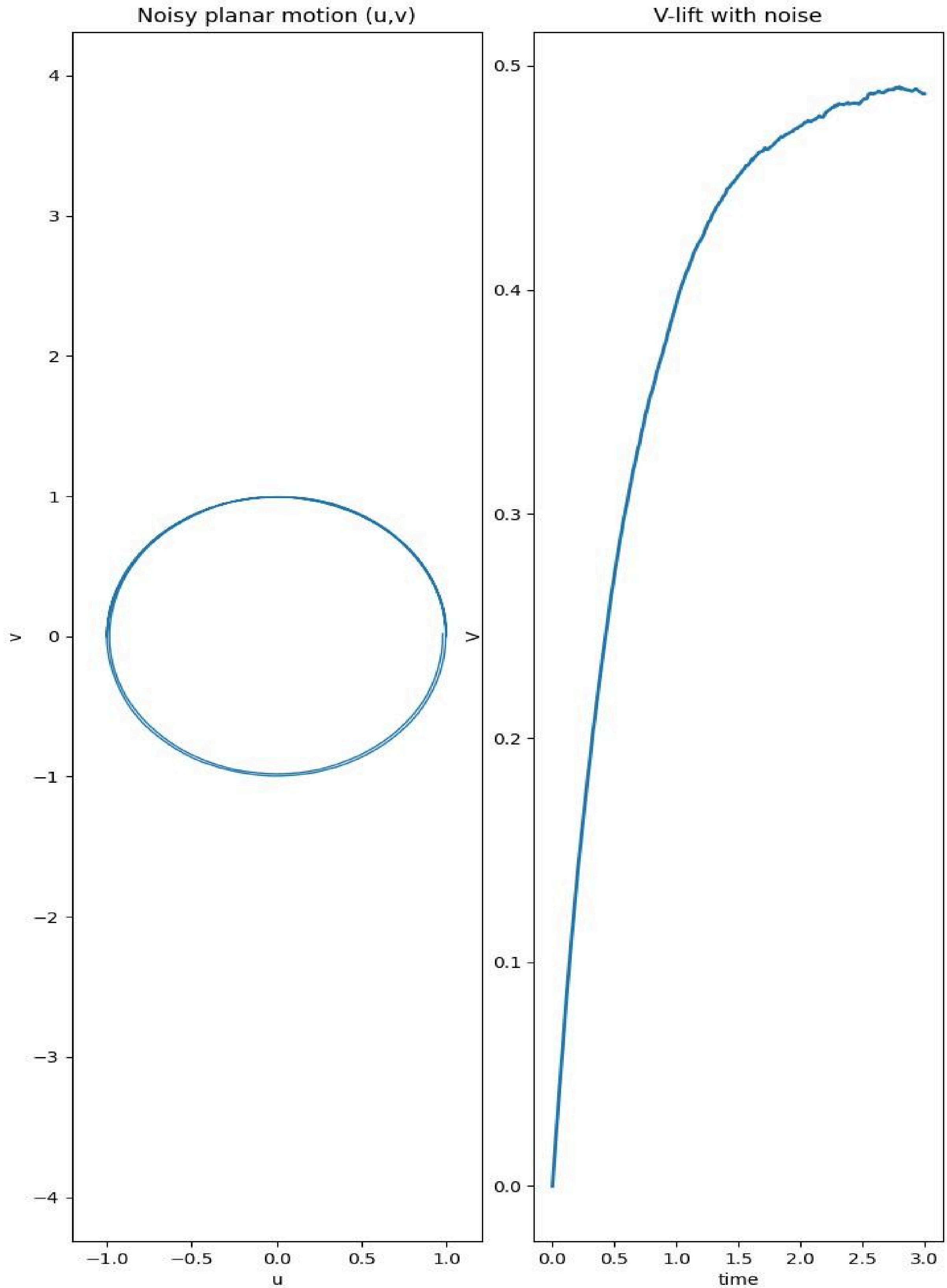
Results

Planar Phase Stability

The planar trajectory remains confined to a narrow annulus around unit radius throughout the braid execution, despite continuous stochastic forcing. No secular growth or decay of the radius is observed, indicating that phase structure is preserved under noise.

Lift-Dimension Convergence

The lift variable exhibits rapid initial growth followed by exponential saturation to a stable equilibrium value. Small stochastic fluctuations persist near the asymptote, consistent with a noise-driven attractor.



Noisy planar motion (u,v)
V-lift with noise
v
u
V
time

Figure 1.1 Noisy planar motion (u,v) (left) and lift-dimension convergence V(t) (right) during execution of the non-Abelian braid.

Interpretation

The coexistence of planar neutrality and lift stabilization demonstrates that the braid is executed as a dynamically attracting process. Deviations introduced by noise are absorbed into the lift channel and dissipated, preventing accumulation of error in the planar phase.

Discussion

These results suggest a reinterpretation of topological protection. Rather than relying solely on abstract equivalence classes, the GSE provides a concrete dynamical mechanism by which topological outcomes are enforced. The braid is not merely preserved; it is actively stabilized.

This perspective bridges topology and control theory, offering a path toward implementing topological operations in realistic noisy hardware without invoking idealized assumptions.

Conclusion

We have shown that non-Abelian braid execution can be stabilized dynamically using the Geometric Schrodinger Equation. By embedding phase evolution within a lift-and-damp geometric flow, braid equivalence emerges as an attractor rather than an assumption. This approach opens new avenues for robust topological quantum operations and suggests that geometry-driven stabilization may play a foundational role in future quantum technologies.

Mathematical Structure of the Geometric Schrodinger Flow

In this section we formalize the mathematical framework underlying the Geometric Schrodinger Equation (GSE) as applied to non-Abelian braid execution under noise. The purpose is to make explicit the dynamical origin of the stabilization observed numerically and to distinguish the mechanism from purely topological or purely unitary constructions.

Extended State Space

The GSE is defined on an extended real state space

$$\Phi(t) = (u(t), v(t), V(t)) \in \mathbb{R}^3,$$

where the planar components (u,v) encode phase-like degrees of freedom and the scalar $V represents a geometric lift variable. Unlike the conventional complex Schrodinger equation, no complex structure is assumed a priori. Instead, rotational behavior emerges from real-valued dynamics.

Define the instantaneous planar radius

$$r(t) = \sqrt{u(t)^2 + v(t)^2}.$$

Deterministic Flow

For a fixed braid generator $\sigma_i^{\pm 1}$, the deterministic GSE flow is given by

$$\dot{u} = -s_i\,\omega\,v,$$
$$\dot{v} = \quad s_i\,\omega\,u,$$
$$\dot{V} = \alpha\,r - kV,$$

where:

- $\omega > 0$ is a rotation rate,
- $s_i \in \{+1, -1\}$ encodes the orientation of the braid generator (σ_i or σ_i^{-1}),
- $\alpha > 0$ controls the strength of geometric lift,
- k > 0 is a damping coefficient.

The planar subsystem (u,v) generates circular orbits with conserved radius in the absence of noise, while the lift equation defines a linear relaxation process driven by the instantaneous planar magnitude.

Stochastic Perturbations

To model realistic control noise, additive stochastic forcing is introduced in all channels:

$$\dot{u} = -s_i\,\omega\,v + \eta_u(t),$$
$$\dot{v} = \quad s_i\,\omega\,u + \eta_v(t),$$
$$\dot{V} = \alpha\,r - kV + \eta_V(t),$$

where η_u, η_v, η_V are independent zero-mean Gaussian noise processes with finite variance.

This defines a stochastic differential system in which stability must arise dynamically rather than through idealized assumptions.

Fixed Point and Attractor Structure

Taking the expectation value over noise realizations and assuming bounded planar motion with $\langle r \rangle \approx r_0$, the lift equation admits a unique stable fixed point

$$V^* = \frac{\alpha r_0}{k}.$$

Linearization about this point yields exponential convergence with rate k, independent of braid length or generator ordering. This fixed point persists under stochastic forcing, producing a noisy attractor rather than a fragile equilibrium.

Braid Execution as Piecewise Flow Composition

A braid word

$$\beta = \sigma_{i_1}^{\epsilon_1} \sigma_{i_2}^{\epsilon_2} \cdots \sigma_{i_n}^{\epsilon_n}, \quad \epsilon_j \in \{\pm 1\},$$

is implemented as a time-ordered composition of flows, each applied over a finite interval Δt with the corresponding sign $s_{i_j} = \epsilon_j.$

The full braid trajectory is therefore a concatenation of stochastic geometric flows:

$$\Phi(T) = \mathcal{F}_{i_n}^{\epsilon_n} \circ \cdots \circ \mathcal{F}_{i_2}^{\epsilon_2} \circ \mathcal{F}_{i_1}^{\epsilon_1}(\Phi(0)),$$

where each $\mathcal{F}_{i_j}^{\epsilon_j}$ denotes the time-Δt flow generated by the GSE.

Emergent Topological Robustness

Crucially, while the planar subsystem alone is neutrally stable, the coupling to the dissipative lift variable renders the full system asymptotically stable in the extended space. Topological equivalence of braid outcomes thus emerges not as an assumption but as a property of the attractor structure of the dynamics.

This mechanism differs fundamentally from traditional topological quantum computing schemes: braid robustness is enforced dynamically rather than postulated abstractly. The observed numerical stability therefore follows directly from the mathematical structure of the GSE.