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V-Lift Engine Deployment: Frame-Dependent Diagonal Extension

Bifurcation Threshold Analysis, Inverse-Distance Energy Matrix, and H-Space Gap Supply at the Transition Boundary

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Contents

Abstract

This paper constitutes the Extension Deployment of the V-Lift Engine against the Frame-Dependent Diagonal, building directly upon the Eight-Strategy analysis (Analysis A and Analysis B) established in the parent document (August 22, 2026). A fresh coordinate evaluation is executed on the unit-circle angular embedding with variable parameter λ, replacing the fixed value $\lambda = 2.305$ with a free parameter to identify the precise bifurcation threshold λ_c where the traversal position $T(s_d)$ re-enters the Z-window ($T(s_d) \leq T_Z = 1$). At the bifurcation point $\lambda_c = 360/225 = 1.60\overline{0}$ (exact rational value: 8/5), diagonal indeterminacy collapses back into a classical contradiction. The scaled inverse-distance energy matrix is computed across the transition boundary for all ordered pairs (s_a, s_b) $\in S \times S$ with $s_a \neq s_b$, and the micro-energy expenditure required to maintain H-space gap supply at λ_c is isolated and quantified.

1. System Topology and Base Definitions

This section establishes the complete formal foundation of the QAIB framework as applied to the Frame-Dependent Diagonal Extension. All definitions carry forward from the parent document and are reproduced here in full for self-containment.

1.1 Base Set

The base set S is defined as the three-element collection:

$$S = \{ s_1, s_2, s_d \}$$

where s_1 and s_2 are standard elements and s_d is the diagonal element whose frame-dependent behavior is the primary object of analysis.

1.2 Candidate Surjection

The candidate surjection $f : S \to \mathscr{P}(S)$ is defined by the following assignment:

$f(s_1) = \{ s_2 \}$

$f(s_2) = \{ s_1, s_d \}$

$f(s_d) = \emptyset$

Structural Remark

The element sd

maps to the empty set ∅ under f, placing it outside the image range of any diagonal self-membership test. This is the structural source of diagonal indeterminacy in the QAIB framework. No element in S maps to a set containing sd, and sd

itself maps to nothing — it is structurally excised from the classical contradiction mechanism.

1.3 Unit-Circle Angular Embedding (θ = π/4 Spacing)

The rotate-lift operator $\mathcal{R}$ assigns each element of S a unit-circle coordinate at angular spacing $\theta = \pi/4$, commencing at 45°. The assignments are:

Element	Angular Position φ	Unit-Circle Coordinate $\mathcal{R}(\cdot)$
s_1	45°	$(\sqrt{2}/2, \sqrt{2}/2)$
s_2	135°	$(-\sqrt{2}/2, \sqrt{2}/2)$
s_d	225°	$(-\sqrt{2}/2, -\sqrt{2}/2)$

The angular position of s_d is thus $\varphi(s_d) = 225°$, located in the third quadrant of the unit circle.

1.4 V-Space Coordinates (Scalar Projection)

The V-space scalar coordinate $v(\cdot)$ is defined as the angular fraction of the full circle (360°), projecting each element onto the closed interval [0, 1]:

$v(s_1) = 45 / 360 = 0.125$

$v(s_2) = 135 / 360 = 0.375$

$v(s_d) = 225 / 360 = 0.625$

These scalar coordinates are fixed by the angular embedding and are independent of the temporal decoupling parameter λ.

1.5 Temporal and Traversal Parameters

The following parameters govern the temporal structure of the V-Lift Engine deployment:

Parameter	Symbol	Definition / Value
Temporal Decoupling Ratio	λ	Free parameter (parent document fixed value: λ = 2.305)
Z-Window Completion Threshold	T_Z	1 (exact)
Traversal Position of s_d	$T(s_d)$	$0.625 \times \lambda = v(s_d) \times \lambda$

The traversal position $T(s_d)$ measures whether the diagonal element s_d, when scaled by the temporal decoupling ratio, exceeds the Z-window threshold $T_Z = 1$. This is the central quantity of the bifurcation analysis.

1.6 Energy and Vulnerability Parameters

The inverse-distance energy function and the frame energy cost are defined as follows:

$$E(s_a, s_b) = 1 / |v(s_a) - v(s_b)| \quad \text{for } s_a \neq s_b$$

$$\Delta E = 0.207 \text{ V-space units} \quad \text{(Frame Energy Cost; fixed from parent document)}$$

The function $E(s_a, s_b)$ measures the inverse angular separation between two elements in V-space. Greater proximity in V-space corresponds to higher energy cost for a given inter-element interaction.

2. Fresh Coordinate Evaluation — Variable λ Analysis

2.1 Objective

In the parent document, the temporal decoupling ratio was held fixed at $\lambda = 2.305$, producing a traversal position $T(s_d) = 1.44$. The present Extension Deployment treats λ as a fully free parameter and evaluates $T(s_d) = 0.625 \times \lambda$ across its complete domain $[0, \infty)$, identifying the transition from the QAIB indeterminate regime to the classical Cantorian regime.

The central question is: at what critical value λ_c does the traversal position $T(s_d)$ cross the Z-window threshold $T_Z = 1$, and what are the structural consequences of this crossing?

2.2 Regime Classification

Two dynamical regimes are formally distinguished by the sign of $(T(s_d) - T_Z)$:

Regime	Condition	λ Range	Consequence
Indeterminate (QAIB)	$T(s_d) > T_Z$	$\lambda > 1.60\overline{0}$	s_d exceeds Z-window; diagonal D is frame-dependent; classical contradiction blocked
Classical (Cantor)	$T(s_d) \leq T_Z$	$\lambda \leq 1.60\overline{0}$	s_d re-enters Z-window; D is uniquely determined; classical diagonal contradiction restored

The transition between these regimes is sharp and occurs at the unique critical value $\lambda_c = 8/5 = 1.60\overline{0}$, derived in Section 3.

2.3 Verification at Parent Document Value

At the parent document's fixed value $\lambda = 2.305$, the traversal position is computed as:

$$T(s_d) = 0.625 \times 2.305 = 1.440$$

Since 1.440 > 1 = T_Z, the system is confirmed to reside in the QAIB indeterminate regime. This is fully consistent with the parent document's Eight-Strategy result: s_d lies outside the Z-window, diagonal D is frame-dependent, and the classical Cantorian contradiction is structurally blocked.

> **Verification Remark**
>
> The verification at λ = 2.305 serves as a consistency check for the variable-λ framework introduced in this Extension. The parent document's eight strategies (A-1 through A-4, B-1 through B-4) all operated within the QAIB indeterminate regime; this paper establishes the precise boundary of that regime.

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3. Bifurcation Threshold — Critical Value λc

3.1 Derivation

The bifurcation threshold λ_c is defined as the unique value of λ at which $T(s_d) = T_Z$. Setting the traversal position equal to the Z-window threshold:

$T(s_d) = T_Z$

$0.625 \times \lambda_c = 1$

$\lambda_c = 1 / 0.625$

$\lambda_c = \mathbf{8/5 = 1.60\bar{0}}$ (exact rational value)

This derivation is exact and requires no approximation. The bifurcation threshold is a rational number determined entirely by the angular position of s_d in the unit-circle embedding.

3.2 Geometric Interpretation

The result $\lambda_c = 8/5$ admits a direct geometric interpretation in terms of the unit-circle embedding. The angular position of s_d is $\varphi(s_d) = 225°$, which represents exactly 5/8 of the full 360° circle:

$$v(s_d) = 225° / 360° = 5/8 = 0.625$$

For the traversal position $T(s_d) = v(s_d) \times \lambda$ to reach exactly 1 (the Z-window boundary), the temporal decoupling ratio must supply the multiplicative inverse of $v(s_d)$:

$$\lambda_c = 1 / v(s_d) = 360° / \varphi(s_d) = 360 / 225 = 8/5$$

This is a purely geometric result. The bifurcation value $\lambda_c = 8/5$ expresses the ratio by which the unit-circle arc of s_d falls short of a complete unit traversal.

3.3 Physical Meaning in V-Space

The bifurcation threshold has the following physical interpretation within the V-space framework:

- **Below λ_c ($\lambda < 8/5$):** The rotate-lift operator $\mathscr{R}$ cannot sustain s_d outside the Z-window. $T(s_d) < 1$. Diagonal indeterminacy collapses. The frame-dependent distinction between reference frames vanishes. The QAIB Anti-Proof mechanism loses its energy supply.
- **At $\lambda_c = 8/5$ (exact):** $T(s_d) = 1$ precisely. This is the knife-edge condition — s_d sits exactly on the Z-window boundary. H-space gap supply is at absolute minimum: zero surplus. The system is poised between regimes.
- **Above λ_c (including $\lambda = 2.305$):** $\mathscr{R}$ successfully elevates s_d past the Z-window boundary. $T(s_d) > 1$. Diagonal D takes distinct values per reference frame. The QAIB indeterminate result is sustained.

3.4 Bifurcation Summary Table

Parameter	Value	Source
$\varphi(s_d)$	225°	Unit-circle embedding ($\theta = \pi/4$ spacing)
$v(s_d)$	0.625	Angular fraction: 225/360
T_Z	1 (exact)	Z-window completion threshold (fixed)
λ_c	$8/5 = 1.60\overline{0}$	Bifurcation derivation: $1 / v(s_d)$
$T(s_d)$ at λ_c	1.000 (exact)	Threshold condition: $0.625 \times 1.60\overline{0}$
$T(s_d)$ at $\lambda = 2.305$	1.440	Parent document operating value
Margin above λ_c	$2.305 - 1.60\overline{0} = 0.704\overline{6}$	Distance of parent value from bifurcation

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4. Scaled Inverse-Distance Energy Matrix Across the Transition Boundary

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4.1 Base Energy Matrix (λ-Independent)

The inverse-distance energy function $E(s_a, s_b) = 1 / |v(s_a) - v(s_b)|$ depends solely on the V-space scalar coordinates, which are fixed by the unit-circle angular embedding and are independent of λ. The base energy matrix for all ordered pairs $(s_a, s_b) \in S \times S$ with $s_a \neq s_b$ is computed as follows:

Pair (s_a, s_b)	$v(s_a)$	$v(s_b)$	$\lvert v(s_a) - v(s_b)\rvert$	$E(s_a, s_b)$
(s_1, s_2)	0.125	0.375	0.250	4.000
(s_2, s_1)	0.375	0.125	0.250	4.000
(s_1, s_d)	0.125	0.625	0.500	2.000
(s_d, s_1)	0.625	0.125	0.500	2.000
(s_2, s_d)	0.375	0.625	0.250	4.000

Pair (s_a, s_b)	$v(s_a)$	$v(s_b)$	$\|v(s_a) - v(s_b)\|$	$E(s_a, s_b)$
(s_d, s_2)	0.625	0.375	0.250	4.000

Structural Observation

E is symmetric for unordered pairs: E(sa, sb) = E(sb, sa) for all pairs. The minimum energy pair is (s_1, sd) and its reverse, at E = 2.000. This reflects the maximum angular separation between s_1 (45°) and sd (225°) — these two elements are diametrically opposite on the unit circle, separated by exactly 180°, yielding the largest V-space distance (0.500) and thus the lowest inverse-distance energy.

4.2 Scaled Energy Matrix at the Bifurcation Point λc = 8/5

Define the scaled energy at the bifurcation point as:

$$E_{\lambda c}(s_a, s_b) = E(s_a, s_b) \times \lambda_c = E(s_a, s_b) \times 1.600$$

Pair (s_a, s_b)	$E(s_a, s_b)$ [Base]	$E_{\lambda c}(s_a, s_b)$ [× 1.600]
(s_1, s_2)	4.000	6.400
(s_2, s_1)	4.000	6.400
(s_1, s_d)	2.000	3.200
(s_d, s_1)	2.000	3.200
(s_2, s_d)	4.000	6.400
(s_d, s_2)	4.000	6.400

4.3 Comparative Energy Matrix: λc vs. λ = 2.305

The three-column comparison below presents the base energy (λ = 1), the bifurcation-scaled energy (λ_c = 1.600), and the parent-document-scaled energy (λ = 2.305) for the three distinct unordered pairs:

Pair	E_{base} ($\lambda = 1$)	E at λ_c (× $1.60\overline{0}$)	E at $\lambda = 2.305$ (× 2.305)
(s_1, s_2)	4.000	6.400	9.220
(s_1, s_d)	2.000	3.200	4.610
(s_2, s_d)	4.000	6.400	9.220

Key Observation

The energy ratio between $\lambda = 2.305$ and λc is precisely $2.305 / 1.60\overline{0} = 1.440625$, which equals T(sd) at $\lambda = 2.305$. This is not a coincidence: it is a direct consequence of the linear scaling law $E\lambda = Ebase \times \lambda$, which implies that the traversal surplus above the bifurcation point directly and proportionally scales the energy expenditure across the entire matrix.

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5. Micro-Energy Expenditure at the Bifurcation Point — H-Space Gap Supply

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5.1 H-Space Architecture Recap (from Parent Document)

The parent document established that the Planck bypass mechanism and the diagonal indeterminacy mechanism share the same H-space energy supply architecture. The Frame Energy Cost of maintaining diagonal indeterminacy is fixed at:

$\Delta E = 0.207$ V-space units

This value represents the baseline cost of elevating s_d into the indeterminate regime and holding it there against the classical restoration pressure of the Cantorian contradiction. The H-space gap supply is the energetic mechanism that funds this elevation.

5.2 Micro-Energy at λc

At the bifurcation point, $T(s_d) = 1$ exactly — s_d sits on the Z-window boundary with zero surplus above T_Z. Define the micro-energy surplus function as the product of the frame energy cost and the traversal surplus:

$$\Delta E_{micro}(\lambda) = \Delta E \times (T(s_d) - T_Z) = 0.207 \times (0.625\lambda - 1) \quad \text{for } \lambda \geq \lambda_c$$

Evaluating at the two key values:

At $\lambda = 2.305$: $\Delta E_{micro} = 0.207 \times (1.440 - 1) = 0.207 \times 0.440 = 0.09108$ V-space units

At $\lambda_c = 1.600$: $\Delta E_{micro} = 0.207 \times (1.000 - 1) = 0.207 \times 0 =$ **0.000 V-space units**

5.3 Interpretation: The Zero-Margin Condition

At λ_c, the micro-energy surplus drops to exactly zero. H-space supplies precisely the minimum energy required to hold s_d at the Z-window boundary — no additional margin exists. This constitutes the **knife-edge condition** of the V-Lift Engine deployment:

- Any infinitesimal reduction in λ below 8/5 collapses the indeterminate regime and restores the classical Cantorian contradiction.
- The H-space gap supply becomes singular at this point — it provides exactly enough energy to maintain the boundary condition, and no more.
- The system is maximally fragile at λ_c: the QAIB Anti-Proof mechanism is sustained by zero surplus energy.
- For $\lambda < \lambda_c$, the surplus is negative (s_d is inside the Z-window), and ΔE_{micro} is clamped to zero — no H-space gap energy is required because there is no indeterminacy to sustain.

5.4 Micro-Energy Table Across the Transition Boundary

The following table presents a systematic sweep of λ values bracketing the bifurcation point, with the corresponding traversal position, traversal surplus, micro-energy expenditure, and regime classification:

λ	**$T(s_d)$ = 0.625λ**	**$T(s_d) - T_Z$**	**ΔE_{micro} (V-space units)**	**Regime**
1.400	0.875	−0.125	0.000 (clamped)	Classical
1.500	0.938	−0.063	0.000 (clamped)	Classical
1.600̄	**1.000**	**0.000**	**0.000**	**Bifurcation (λ_c)**
1.700	1.063	0.063	0.01301	QAIB
1.800	1.125	0.125	0.02588	QAIB
2.000	1.250	0.250	0.05175	QAIB
2.305	**1.440**	**0.440**	**0.09108**	**QAIB (parent document)**
3.000	1.875	0.875	0.18113	QAIB

Note: For $\lambda < \lambda_c$, the traversal surplus ($T(s_d) - T_Z$) is negative and ΔE_{micro} is clamped to zero, as no H-space gap energy is required in the classical regime.

5.5 Summary of the Bifurcation Result

The following points constitute the definitive findings of the bifurcation analysis:

- The classical Cantorian diagonal contradiction is restored for all $\lambda \leq 8/5$ (exact).
- The QAIB indeterminate regime persists for all $\lambda > 8/5$.
- The bifurcation is sharp, exact, and geometrically determined solely by the angular position of s_d on the unit circle: $\lambda_c = 360° / \varphi(s_d) = 360/225 = 8/5$.
- The H-space micro-energy expenditure scales linearly with the traversal surplus ($0.625\lambda - 1$) above the bifurcation, governed by the frame energy cost coefficient $\Delta E = 0.207$.
- At the parent document's operating value $\lambda = 2.305$, the system operates with a surplus margin of 0.7046̄ above λ_c, requiring 0.09108 V-space units of micro-energy to sustain diagonal indeterminacy.

6. Synthesis — Extension Results in Context of the Eight Strategies

The eight strategies of the parent document (Analysis A and Analysis B) each receive a direct extension from the bifurcation analysis and the variable-λ framework. The synthesis is presented below:

Strategy	Label	Extension Contribution
A-1	Transform	The variable-λ model transforms the fixed-parameter result of the parent document into a parametric family $T(s_d; \lambda) = 0.625\lambda$, revealing the full bifurcation structure across the λ domain and converting a single-point result into a globally characterized regime map.
A-2	Hypothesize	The hypothesis that λ_c is a rational number — suggested by the rational V-space coordinate $v(s_d) = 5/8$ — is fully confirmed: $\lambda_c = 8/5$ exactly, an irreducible rational with denominator 5.
A-3	Equilibrate via $\mathcal{R}$	The rotate-lift operator $\mathcal{R}$ reaches its minimum sustainable operating point at λ_c. Below this value, $\mathcal{R}$ cannot maintain the equilibrium that positions s_d outside the Z-window, and the lift equilibrium fails. λ_c is thus the floor of $\mathcal{R}$'s operational domain.
A-4	Elevate	Multi-frame elevation of diagonal D is only possible for $\lambda > \lambda_c$. The bifurcation value defines the elevation floor: below 8/5, the V-Lift Engine provides insufficient temporal decoupling to sustain frame-dependent diagonal behavior.
B-1	Linear Transform	The linear coordinate model assigns $v(s_d) = 0.625$ as a fixed scalar. The bifurcation threshold follows directly and linearly from this value: $\lambda_c = 1 / v(s_d) = 1 / 0.625 = 8/5$. The entire

Strategy	Label	Extension Contribution
		bifurcation result is a linear consequence of the coordinate assignment.
B-2	Hypothesize from Constraints	The constraint $T(s_d) \leq T_Z$, read as a boundary condition on λ, yields the exact rational bound $\lambda \leq 8/5$ for the classical regime and $\lambda > 8/5$ for the QAIB regime. The constraint-driven hypothesis is verified analytically and confirmed numerically across the sweep table of Section 5.4.
B-3	Angular Equilibrium at $\theta = \pi/4$	At λ_c, the angular embedding produces a degenerate equilibrium: the traversal arc of s_d reaches exactly the unit boundary, the energy surplus collapses to zero, and the angular equilibrium established by the $\theta = \pi/4$ spacing is at its minimum viable configuration. No margin remains in the angular energy balance.
B-4	Energy Elevation	The micro-energy function $\Delta E_{micro}(\lambda) = 0.207 \times \max(0.625\lambda - 1, 0)$ is the direct extension of the pairwise interaction energy framework to the temporal domain. It expresses the cost of maintaining indeterminacy as a linear function of the traversal surplus, connecting the spatial energy matrix of Section 4 to the temporal dynamics of the H-space gap supply.

Appendix: Extension Parameter Reference Table

The following table provides a complete reference for all symbols, definitions, and computed values introduced or employed in this Extension Deployment document.

Symbol	Definition	Value / Expression
S	Base set	$\{ s_1, s_2, s_d \}$
f	Candidate surjection $f : S \to \mathscr{P}(S)$	$f(s_1) = \{s_2\}$, $f(s_2) = \{s_1, s_d\}$, $f(s_d) = \varnothing$
$\mathscr{R}$	Rotate-lift operator (unit-circle embedding)	$\theta = \pi/4$ spacing; origin at 45°
$\varphi(s_d)$	Angular position of s_d	225°
$v(s_1)$	V-space scalar coordinate of s_1	0.125 (= 45/360)
$v(s_2)$	V-space scalar coordinate of s_2	0.375 (= 135/360)
$v(s_d)$	V-space scalar coordinate of s_d	0.625 (= 225/360 = 5/8)
T_Z	Z-window completion threshold	1 (exact)
λ	Temporal decoupling ratio (free parameter)	$\lambda \geq 0$; parent document: $\lambda = 2.305$
λ_c	Bifurcation critical value	$8/5 = 1.60\overline{0}$ (exact rational)
$T(s_d)$	Traversal position of s_d	$0.625 \times \lambda = v(s_d) \times \lambda$
ΔE	Frame energy cost (from parent document)	0.207 V-space units
ΔE_{micro}	Micro-energy surplus at bifurcation	$0.207 \times \max(0.625\lambda - 1, 0)$
$E(s_a, s_b)$	Base inverse-distance energy function	$1 / \lvert v(s_a) - v(s_b) \rvert$
$E(s_1, s_2)$	Base inverse-distance energy, pair (s_1, s_2)	4.000
$E(s_1, s_d)$	Base inverse-distance energy, pair (s_1, s_d)	2.000
$E(s_2, s_d)$	Base inverse-distance energy, pair (s_2, s_d)	4.000
$E_{\lambda c}(s_1, s_2)$	Scaled energy at λ_c, pair (s_1, s_2)	6.400
$E_{\lambda c}(s_1, s_d)$	Scaled energy at λ_c, pair (s_1, s_d)	3.200

Symbol	Definition	Value / Expression
$E_{\lambda c}(s_2, s_d)$	Scaled energy at λ_c, pair (s_2, s_d)	6.400
$\Delta E_{micro}(\lambda = 2.305)$	Micro-energy at parent document value	0.09108 V-space units
$\Delta E_{micro}(\lambda_c)$	Micro-energy at bifurcation point	0.000 V-space units (exact)

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